## STAT 462/662 Introduction to Stochastic Processes

## Homework 1

## Due: Tuesday September 7, 2021

## \*Show all your work to receive full credit.

- 1. Let X be a Poisson random variable with parameter  $\lambda$ . Use the definition of expected value to calculate E[X].
- 2. An urn contains two red, two orange, and two blue balls. Two balls are randomly selected. The outcomes R, O, and B denote that the color of the selected ball is red, orange, and blue, respectively.
  - (a) What is the sample space of this experiment?
  - (b) Let X represent the number of orange balls selected. What are the possible values of X?
  - (c) Calculate P(X = 0).
- 3. We toss a fair coin four times. Let X represent the number of heads obtained.
  - (a) What are the possible values of X?
  - (b) What are the probabilities associated with the values that X can take on?
- 4. The joint probability mass function for random variables X and Y is given below, where p(x, y) = P(X = x, Y = y). Compute E[X|Y = i] for i = 1, 2, 3.

$p(1,1) = \frac{1}{9},$	$p(2,1) = \frac{1}{3},$	$p(3,1) = \frac{1}{9}$
$p(1,2) = \frac{1}{9},$	p(2,2) = 0,	$p(3,2) = \frac{1}{18}$
p(1,3) = 0,	$p(2,3) = \frac{1}{6},$	$p(3,3) = \frac{1}{9}$

5. A Markov chain  $X_0, X_1, \ldots$  with state space  $S = \{0, 1, 2\}$  has transition probability matrix

$$\mathbf{P} = \left(\begin{array}{rrrr} 0.1 & 0.3 & 0.6\\ 0.7 & 0.2 & 0.1\\ 0.2 & 0.3 & 0.5 \end{array}\right)$$

and initial distribution  $P(X_0 = 0) = 0.4$ ,  $P(X_0 = 1) = 0.5$ , and  $P(X_0 = 2) = 0.1$ . Compute the probability  $P(X_0 = 0, X_1 = 1, X_2 = 2)$ .