

STAT 467/667 Homework 2
SPRING 2015

Due in class on Friday February 6.

1. (**Stat 467 only**) Suppose that X and Y have a continuous joint distribution for which the joint PDF is as follows:

$$f(x, y) = \begin{cases} \left(\frac{3}{2}\right) y^2 & \text{for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Determine the marginal PDFs of X and Y .
(b) Determine the conditional PDFs of X given $Y = y$ and of Y given $X = x$.
(c) Compute the probability $P(X < 1|Y = 0.5)$.
2. (**Stat 667 only**) Suppose that X and Y have a continuous distribution for which the joint PDF is as follows:

$$f(x, y) = \begin{cases} \left(\frac{15}{4}\right) x^2 & \text{for } 0 \leq y \leq 1 - x^2, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Determine the marginal PDFs of X and Y .
(b) Are X and Y independent?
(c) Determine the conditional PDFs of X given $Y = y$ and of Y given $X = x$.
3. Suppose there are 4 red and 6 green marbles in a box, and two are taken out with no replacement. Let X_1 and X_2 be indicator random variables associated with selecting a red marble, i.e., $X_i = 1$ if the i^{th} marble selected is red and 0 otherwise.
- (a) Determine the joint PDF of X_1 and X_2 .
(a) Determine the marginal PDFs of X_1 and X_2 .
(b) Are X_1 and X_2 independent?
(c) Determine the conditional PDFs of X_1 given $X_2 = 1$ and of X_2 given $X_1 = 0$.
4. What is the probability that the larger of two random observations drawn from any continuous PDF will exceed the 60th percentile?

5. Let X_1, \dots, X_n be a random sample of size n from a probability distribution given by the PDF f . For each of the following choices of f determine the joint PDF of the random vector $X = (X_1, X_2, \dots, X_n)$.
- (a) f if the pdf of the Poisson distribution with parameter (mean) λ .
 - (b) f is the pdf of the Bernoulli distribution with parameter p .
 - (c) f is the pdf of the geometric distribution with parameter p .
 - (d) f is the pdf of the gamma distribution with parameters α and β .
6. Let X_1, \dots, X_n be a random sample of size n from a probability distribution given by the PDF f . For each of the following choices of f determine the joint PDF of the sum $Y = X_1 + \dots + X_n$. [**Hint:** Use the moment generating function].
- (a) f if the pdf of the Poisson distribution with parameter (mean) λ .
 - (b) f is the pdf of the Bernoulli distribution with parameter p .
 - (c) f is the pdf of the geometric distribution with parameter p .
 - (d) f is the pdf of the gamma distribution with parameters α and β .