STAT 467/667 Homework 3

SPRING 2015

Due in class on Friday February 13. Make sure to note whether the question is asking for the estimate θ_e or the estimator $\hat{\theta}$.

1. **Chapter 5, No. 5.2.1 (Stat 467 only)**

   A random sample of size 8: $X_1 = 1, X_2 = 0, X_3 = 1, X_4 = 1, X_5 = 0, X_6 = 1, X_7 = 1, X_8 = 0$ is taken from the probability function

   $$p_X(k \mid \theta) = \theta^k (1-\theta)^{1-k}, \quad k = 0, 1; \quad 0 < \theta < 1$$

   Find the maximum likelihood estimate for θ.

2. **Chapter 5, No. 5.2.3**

   Use the sample $Y_1 = 8.2, Y_2 = 9.1, Y_3 = 10.6, Y_4 = 4.9$ to calculate the maximum likelihood estimate for λ in the exponential PDF

   $$f_Y(y \mid \lambda) = \lambda e^{-\lambda y}, \quad y \geq 0$$

3. **Chapter 5, No. 5.2.4**

   Suppose a random sample of size n is drawn from the probability model

   $$p_X(k \mid \theta) = \frac{\theta^{2k} e^{-\theta^2}}{k!}, \quad k = 0, 1, 2, ...$$

   Find a formula for the maximum likelihood estimator, $\hat{\theta}$.

4. **Chapter 5, No. 5.2.6**

   Use the method of maximum likelihood to estimate θ in the PDF

   $$f_Y(y \mid \theta) = \frac{\theta}{2\sqrt{y}} e^{-\theta\sqrt{y}}, \quad y > 0$$

   Evaluate θ_e for the following random sample of size 4: $Y_1 = 6.2, Y_2 = 7.0, Y_3 = 2.5, Y_4 = 4.2$.

5. **Chapter 5, No. 5.2.10 (Stat 667 only)**

   Find the maximum likelihood estimate for θ in the PDF

   $$f_Y(y \mid \theta) = \frac{2y}{1-\theta^2}, \quad \theta \le y \le 1$$

   if a random sample of size 6 yielded the measurements 0.70, 0.63, 0.92, 0.86, 0.43, 0.21.

6. **Chapter 5, No. 5.2.12**

   If the random variable Y denotes an individual's income, Pareto's law claims that $P(Y \ge y) = \left(\frac{k}{y}\right)^\theta$, where k is the entire population's minimum income. It follows that $F_Y(y) = 1 - \left(\frac{k}{y}\right)^\theta$, and by differentiation,

   $$f_Y(y \mid \theta) = \theta k^\theta \left(\frac{1}{y}\right)^{\theta+1}, \quad y \ge k, \quad \theta \ge 1$$

   Assume k is known. Find the maximum likelihood estimator for θ if income information has been collected on a random sample of 25 individuals.

7. **Chapter 5, No. 5.2.15**

   Let $y_1, y_2, ..., y_n$ be a random sample of size n from the uniform PDF, $f_Y(y \mid \theta) = \frac{1}{\theta}, 0 \le y \le \theta$. Find a formula for the method of moments estimate (MOM) for θ. Compare the values of the MOM estimate and the maximum likelihood estimate if a random sample of size 5 consists of the numbers 17, 92, 46, 39, and 56.

   (**Hint:** See your class notes from 2/6/15 for the MLE!)

8. **Chapter 5, No. 5.2.16**

   Use the method of moments to estimate θ in the PDF

   $$f_Y(y \mid \theta) = (\theta^2 + \theta) y^{\theta-1}(1-y), \quad 0 < y < 1$$

   Assume that a random sample of size n has been collected.

9. **BONUS QUESTION: Chapter 5, No. 5.2.9 part (b)**

   Suppose the random sample $Y_1 = 6.3, Y_2 = 1.8, Y_3 = 14.2, Y_4 = 7.6$ represents the two-parameter uniform PDF

   $$f_Y(y \mid \theta) = \frac{1}{\theta_2 - \theta_1}, \quad \theta_1 \le y \le \theta_2$$

   Find the maximum likelihood estimates for θ_1 and θ_2.