

STAT 467/667 Homework 6

SPRING 2015

Due in class on Friday March 13. These problems refer to the 4<sup>th</sup> Edition.

1. Chapter 5, No. 5.8.1
2. Chapter 5, No. 5.8.7
3. Chapter 5, No. 5.8.8
4. Suppose that  $X_1, X_2, \dots, X_n$  are *i.i.d* Bernoulli random variables with parameter  $\theta$ . That is, each random variable has PDF  $f_X(x | \theta) = \theta^x(1 - \theta)^{1-x}$  for  $x = 0, 1$ . Further, assume that the prior distribution of  $\theta$  is the uniform PDF on  $[0, 1]$ .
  - (a) Compute the posterior distribution of  $\theta$  given the data  $X_1, X_2, \dots, X_n$ .
  - (b) Compute the Bayes estimate of  $\theta$  assuming the squared error loss function  $L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$ .
  - (c) Illustrate the above calculations when  $n = 100$  and  $\sum_{i=1}^n X_i$  is 0, 50, and 100. Write a few sentences describing the effect of data on the posterior distribution and the Bayes estimate.
5. **(BONUS QUESTION)** How does the posterior distribution change in Problem 4 if we assume that the prior distribution of  $\theta$  is the beta PDF with parameters  $r$  and  $s$ ? Explain how Problem 4 is a special case of this one.