STAT 467/667 PRACTICE PROBLEMS FOR EXAM 1

SPRING 2015

Instructions: Please show all your work and justify your answers!

1. Suppose that X and Y have a continuous joint distribution for which the joint PDF is as follows:

$$f_{X,Y}(x,y) = \begin{cases} \frac{3}{2}y^2 & \text{for } 0 \le x \le 2 \text{ and } 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Determine the marginal PDF of Y.

   (b) Compute the probability $P(X < 1 \mid Y = 0.25)$.

2. Suppose $X_1, \ldots, X_n$ are *i.i.d.* random variables from the distribution having the following probability density function:

$$f_X(x \mid \theta) = \theta x^{\theta - 1}, \;\; 0 < x < 1, \;\; 0 < \theta < \infty.$$

   Find an estimator of θ using the method of moments.

3. Suppose $X_1, \ldots, X_n$ are *i.i.d.* random variables from the uniform distribution on the interval $[0, \theta]$, and let $X_{(n)}$ be the largest order statistic (the maximum of the X_i's). What constant c makes $cX_{(n)}$ an unbiased estimator of θ?

4. A random sample $X_1, \ldots, X_n$ is taken from a normal distribution with an unknown mean μ and **given** variance σ^2. If $\sigma^2 = 12$, what is the smallest sample size for which the length of the 95% confidence interval for μ is less than or equal to 5?

5. Let $X_1, ..., X_n$ be a random sample from the exponential distribution with PDF $f_X(x \mid \theta) = \frac{1}{\theta}e^{-x/\theta},\ x > 0,\ \theta > 0$.

   (a) Find the maximum likelihood estimator of θ.

   (b) Is the MLE you found in part (a) a consistent estimator of θ? Why or why not?

   (c) Given that $\hat{\theta}_1 = X_1$ and $\hat{\theta}_2 = \bar{X}$ are both unbiased estimators for θ, which one is more efficient? Justify your answer.

Some useful formulas - might want to include these in your formula sheet.

- Maximum order statistic. Let $X_1, X_2, ..., X_n$ be a random sample from PDF f and CDF F. The CDF F_{X_n} and the PDF f_{X_n} of the largest order statistic $X_{(n)}$ are

$$F_{X_n}(x) = (F(x))^n \text{ and } f_{X_n}(x) = n(F(x))^{n-1}f(x).$$

- The mean of the exponential distribution with PDF $f_X(x \mid \theta) = \frac{1}{\theta}e^{-x/\theta},\ x > 0,\ \lambda > 0$ is θ. The variance of this distribution is θ^2.

- $(1 - \alpha)100\%$ CI for the mean of a Normal distribution with known variance σ^2 is

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$