Numerically solving SDEs (con't)

Look at an example of Runge-Kutta 4 algorithm

- 4^{th} order method (higher order Taylor approx)
- more accurate than Euler's method (only 1^{st} order accurate) linear

Ref: Paul Hurtado's book chapter (2020)
§3.4 SDEs

↳ Also posted this on Canvas: SDEs_Hurtado.pdf

↳ R code on class website: sde-runge-kutta.r

Example: Stochastic SIR Model

First, ODE model of SIR model for infectious disease

S = Susceptible
I = infected
R = recovered

total population size is N indivs.
$N = S + I + R$

$$\begin{cases} \dfrac{dS(t)}{dt} = -\lambda(t)S(t) \\[2ex] \dfrac{dI(t)}{dt} = \lambda(t)S(t) - \gamma I(t) \\[2ex] \dfrac{dR(t)}{dt} = \gamma I(t) \end{cases}$$

where $\lambda(t) = \beta I(t)$ per capita infection rate

γ = per capita recovery rate

$$\boxed{S} \rightarrow \boxed{I} \rightarrow \boxed{R}$$

This ODE model can be viewed as the "mean field" model for an underlying stochastic state transition model of a large (but finite) # of individuals,

- each transition from state S to I according to event times of a NHPP first event time dist'n with rate $\lambda(t)$;

- each indiv moves $I \to R$ following HPP first event time dist'n with rate γ.

  $\Rightarrow$ amount of time an indiv. spends infected follows an exponential dist'n with mean $\frac{1}{\gamma}$.

Indiv-based model

SDE approximation of stochastic SIR Model

$$dX(t) = \mu(X(t))\,dt + B(X(t))\,dW(t)$$

where $X(t) = \begin{bmatrix} S(t) \\ I(t) \end{bmatrix}$ & $R(t) = N - S(t) - I(t)$

2 Dim vector

$$\underbrace{\begin{bmatrix} dS(t) \\ dI(t) \end{bmatrix}}_{dX} = \underbrace{\begin{bmatrix} -\lambda(t)S(t) \\ \lambda(t)S(t) - \gamma I(t) \end{bmatrix}}_{\mu \text{ (drift coeff)}} dt + B\,dW(t)$$

see Paul's chapter Eq 59 or R code for details (diffusion coeff.)