

STAT 753 Homework 5
SPRING 2020

Due on Thursday April 16 (anytime). Problems are from the Probability Models book.

1. Chapter 5, Problem 56
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3. Simulate a nonhomogeneous Poisson process with intensity function

$$\lambda(t) = 6 + 5 \cos(\pi t).$$

- (a) Plot the intensity function as a function of time.
 - (b) Generate a two independent sample paths and plot on the same figure for the time interval $[0,10]$.
 - (c) Compute the average value of $\lambda(t)$ over $[0,10]$ and simulate a **homogeneous** Poisson process with this average rate.
 - (d) Generate a sample path of the HPP to compare with the NHPP version above (include a figure).
 - (e) Briefly discuss your observations.
4. Let $N(t)$ be a Poisson process with rate λ , and let Y_1, Y_2, \dots be *i.i.d.* random variables with mean μ and variance σ^2 . Derive the mean and the variance of the compound Poisson random variable

$$Z(t) = \sum_{i=1}^{N(t)} Y_i.$$

5. Give an example of a compound Poisson process $\{Z(t) : t \geq 0\}$ where $Z(t)$ is defined as in Problem 4. Make sure to explain what $N(t)$, Y_i , and $Z(t)$ stand for in the example.
6. Write an algorithm in R to simulate a linear birth and death process such that the total birth rate is $\lambda_i = i\lambda$ and the total death rate is $\mu_i = i\mu$. Use $\lambda = 1$, $\mu = 0.5$, and start with 100 individuals in the population.
- (a) Generate three sample paths of the process and include a figure (for time interval $[0,100]$).
 - (b) What happens if the death rate is increased to $\mu = 2$? Include another figure with three sample paths for this case.
 - (c) Briefly describe your observations of the processes in (a) and (b).
 - (d) **BONUS:** Modify the algorithm above to include immigration at an exponential rate θ . Generate sample paths for 3 different values of θ , include R code, parameter values, and a figure.

Include your R code along with the answers to the items listed above.